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Stability of Paracontractive Open Multi-Agent Systems

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Outline

1. Introduction
2. Background on open multi-agent systems (with a running example)
3. Main Result: convergence of paracontractive OMASs
4. Numerical simulations
5. Conclusions

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Figure: Multi-Robot Systems

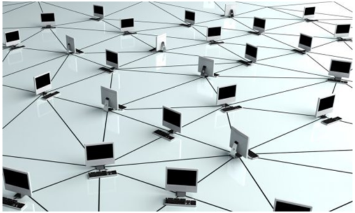

Figure: Peer-to-Peer Networks

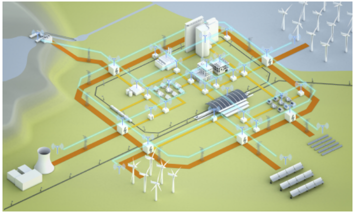

Figure: Smart grids

Figure: Distributed learning

A small sample of the literature on Open MAS

A growing interest in our community:

- M. Abdelrahim, J. M. Hendrickx, and W. Heemels, "*Max consensus in open multi-agent systems with gossip interactions*", in IEEE 56th Annual Conference on Decision and Control (2017).
- V. S. Varma, I.-C. Morarescu, and D. Nesic, "*Open multi-agent systems with discrete states and stochastic interactions*", in IEEE Control Systems Letters (2018).
- M. Franceschelli and P. Frasca, "*Stability of open multiagent systems and applications to dynamic consensus*", in IEEE Trans actions on Automatic Control (2020).
- Y.-G. Hsieh, F. Iutzeler, J. Malick, and P. Mertikopoulos, "*Optimization in Open Networks via Dual Averaging*", in 60th IEEE Conference on Decision and Control (2021).
- Z. A. Z. S. Dashti, G. Oliva, C. Seatzu, A. Gasparri, and M. Franceschelli, "*Distributed mode computation in open multi agent systems*", in IEEE Control Systems Letters (2022).
- N. Hayashi, "*Distributed Subgradient Method in Open Multiagent Systems*", IEEE Transactions on Automatic Control (2023).
- C. M. d. Galland, R. Vizuete, J. M. Hendrickx, E. Panteley, and P. Frasca, "*Random coordinate descent for resource allocation in open multi-agent systems*", in IEEE Transactions on Automatic Control (2024).
- ...

Our contribution

Contribution 1

Formulation of general stability criteria for the class of "*paracontractive*" open multi-agent systems, that is a superclass of those being "*contractive*".

Contribution 2

Presentation of a novel algorithm to solve the "*max-tracking problem*", also knwon as the "*dynamic max-consensus problem,*" in Open MAS which we show it falls into the class of "*paracontractive*" open multi-agent systems

Diego Deplano, Mauro Franceschelli, Alessandro Giua "Dynamic Min and Max Consensus and Size Estimation of Anonymous Multi-Agent Networks", IEEE Transactions on Automatic Control, 2023

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Problem set-up

Undirected network → $\mathcal{G}_k = (\mathcal{V}_k, \mathcal{E}_k)$

Set of agents → $\mathcal{V}_k = \{1, \ldots, n_k\}$

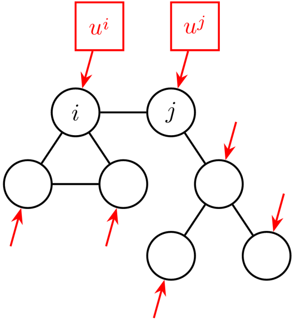
Set of interactions → $\mathcal{E}_k \subseteq \mathcal{V}_k \times \mathcal{V}_k$

State of agent i → $x_k^i \in \mathbb{R}$

Reference signal of agent i → $u_k^i \in \mathbb{R}$

Neighbors of agent i → $\mathcal{N}_k^i = \{j | (i, j \in \mathcal{V}_k) \in \mathcal{E}_k\}$

Framework → Discrete-time $k \in \mathbb{N}$



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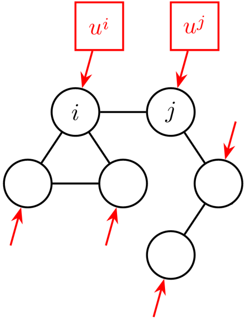

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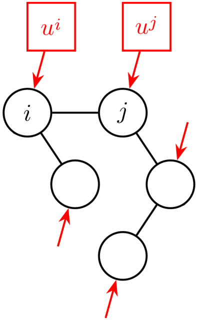

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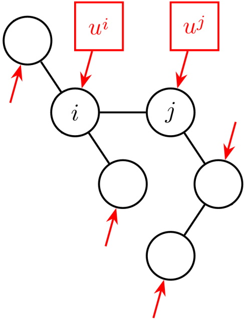

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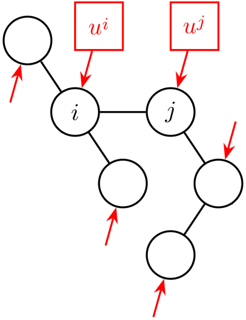
Framework → Discrete-time $k \in \mathbb{N}$

Remaining agents → $\mathcal{R}_k = \mathcal{V}_k \cap \mathcal{V}_{k-1}$

Arriving agents → $\mathcal{A}_k = \mathcal{V}_k \setminus \mathcal{V}_{k-1}$

Departing agents → $\mathcal{D}_k = \mathcal{V}_k \cap \mathcal{V}_{k+1}$



$$x_k^i = \begin{cases} f^i(x_{k-1}, u_k, \mathcal{G}_{k-1}) & \text{if } i \in \mathcal{R}_k, \\ h^i(u_k^i) & \text{if } i \in \mathcal{A}_k, \end{cases} \quad k \in \mathbb{N} \setminus \{0\}, \tag{1}$$

A running example: dynamic-max consensus in open networks (1)

The basic version of the dynamic max-consensus (DMC) protocol for open networks, called "ODMC" is:

$$x_k^i = \begin{cases} \max\limits_{j \in \mathcal{N}_{k-1}^i} \left\{ x_{k-1}^j - \alpha, u_k^i \right\} & \text{if } i \in \mathcal{R}_k, \\ u_k^i & \text{if } i \in \mathcal{A}_k, \end{cases}$$

where $u_k^i \in \mathbb{R}$ are scalar time-varying signals, and denote

$$\overline{u}_k = \max_{i \in \mathcal{V}_k} u_k^i.$$

Notions of open multi-agent systems – Trajectory of points of interest

Let g_k denote the time-varying map ruling the "standard dynamics" when no agent joins/leave:

$$x_k = g_k(x_{k-1}) := f(x_{k-1}, u_k, \mathcal{G}_{k-1}), \text{ when } \mathcal{V}_k = \mathcal{V}_{k-1}, \tag{2}$$

Question: How the notion of "equilibrium point" translates for open systems?

Definition: Trajectory of points of interest

Consider an OMAS and assume that the standard dynamics has a unique solution $\hat{x}_k$ at each time k,

$$\hat{x}_k = g_{k+1}(\hat{x}_k).$$

The (open) sequence $\{\hat{x}_k : k \in \mathbb{N}\}$ is called the "trajectory of points of interest" (TPI) of the OMAS.

Remark: For autonomous and closed system, this definition boils down to that of an equilibrium point.

M. Franceschelli and P. Frasca, "*Stability of open multiagent systems and applications to dynamic consensus*", in IEEE Transactions on Automatic Control (2020).

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A running example: dynamic-max consensus in open networks (2)

Consider the ODMC protocol:

$$x_k^i = \begin{cases} \max\limits_{j \in \mathcal{N}_{k-1}^i} \left\{ x_{k-1}^j - \alpha, u_k^i \right\} & \text{if } i \in \mathcal{R}_k, \\ u_k^i & \text{if } i \in \mathcal{A}_k. \end{cases}$$

The point of interest $\hat{x}_k$ at each time k is given component-wise by

$$\hat{x}_k^i = \overline{u}_k - \alpha \pi_k^i$$

where π_k^i denotes the distance (number of edges) of node i from the (closest) node with the maximum signal $\overline{u}_k$.

Remark: The TPI consists of points $\hat{x}_k = [\cdots, \hat{x}_k^i, \dots]^\top \in \mathbb{R}^{|\mathcal{V}_k|}$ of different dimension since $i \in \mathcal{V}_k$.

Notions of open multi-agent systems

Question: When does an OMAS admit a TPI?

Definition: Paracontractivity

Let $\Gamma \geq 0, T \geq 1$. An OMAS is said to be "(Γ, T)-paracontractive" w.r.t. $\|\cdot\|_\infty$ if there exists $\gamma \in [0,1)$ such that for all $k \geq 0$ and for all $x \in \mathbb{R}^{m|\mathcal{V}_k|}$ it holds

$$\|(g_{k+T} \circ \cdots \circ g_{k+1})(x) - \hat{x}_k\|_\infty \leq \max\{\gamma \|x - \hat{x}_k\|_\infty, \Gamma\}, \quad (3)$$

where $\{\hat{x}_k : k \in \mathbb{N}\}$ is the TPI of the OMAS, and $\mathcal{V}_k = \cdots = \mathcal{V}_{k+T-1}$.

Remarks:

- Contractivity is a special case of paracontractivity.
- When $T > 1$ the system may be nonexpansive at each time step, but it contracts after T steps.

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A running example: dynamic-max consensus in open networks (3)

Consider the ODMC protocol:

$$x_k^i = \begin{cases} \max\limits_{j \in \mathcal{N}_{k-1}^i} \left\{ x_{k-1}^j - \alpha, u_k^i \right\} & \text{if } i \in \mathcal{R}_k, \\ u_k^i & \text{if } i \in \mathcal{A}_k. \end{cases}, \qquad \text{and assume} \qquad |u_k^i - u_{k-1}^i| \leq \Pi.$$

Consider the following worst case:

- Line graph
- All inputs are equal
- One node has value equal to the input
- All other nodes have state value much below the input

Conclusions:

- It will take at most $\delta_k + 1$ steps (the graph's diameter) to see a contraction: $T \geq \delta_k + 1$.
- The step size α must be grater than the maximum rate of change Π of the reference signal: $\alpha > \Pi$.
- It cannot contract if it is already too close to the TPI: $\Gamma \geq (\delta_k + 1)\alpha$.

Notions of open multi-agent systems – Stability of the TPI

Definition: Open stability

Consider an OMAS with state evolution $\{x_k : k \in \mathbb{N}\}$. Its TPI $\{\hat{x}_k : k \in \mathbb{N}\}$ is said to be "open stable" w.r.t. $\|\cdot\|_\infty$ if there is a stability radius $R \geq 0$ with the following property: for every $\varepsilon > R$, there exists $\delta > 0$ such that:

$$\|x_0 - \hat{x}_0\|_\infty < \delta \Rightarrow \|x_k - \hat{x}_k\|_\infty < \varepsilon, \quad \forall k \geq 0.$$

Remark: The infinity norm $\|\cdot\|_\infty$ allows for a fair comparison of distances evaluated in spaces of different dimensions.

Definition: Global asymptotic open stability

Consider an OMAS whose TPI $\{\hat{x}_k : k \in \mathbb{N}\}$ is open stable with stability radius $R \geq 0$. The TPI is said to be "globally asymptotically open stable" w.r.t. $\|\cdot\|_\infty$ if all trajectories converge to within a distance of R from the TPI:

$$\limsup_{k \to \infty} \|x_k - \hat{x}_k\|_\infty \leq R.$$

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Sufficient conditions for global asymptotic open stability

Definition: Bounded TPI

The TPI $\{\hat{x}_k : k \in \mathbb{N}\}$ of an OMAS is said to have "bounded variation" if

$$\exists B \geq 0 : \quad \max_{r \in \mathcal{R}_k} \|\hat{x}_k^r - \hat{x}_{k-1}^r\|_\infty \leq B, \quad \forall k \in \mathbb{N}.$$

Definition: Slow nonexpansiveness

Let $\Lambda \geq 0$. An OMAS is said to be "Λ-slowly expansive" w.r.t. $\|\cdot\|_\infty$ if for all $k \geq 0$ and for all $x \in \mathbb{R}^{m|\mathcal{V}_k|}$ it holds

$$\|g_{k+1}(x) - \hat{x}_k\|_\infty \leq \|x - \hat{x}_k\|_\infty + \Lambda, \tag{4}$$

where $\{\hat{x}_k : k \in \mathbb{N}\}$ is the TPI of the OMAS.

Definition: Bounded arrival process

The arrival process of an OMAS with TPI $\{\hat{x}_k : k \in \mathbb{N}\}$ is said to be "bounded" if

$$\exists H \geq 0 : \quad \max_{a \in \mathcal{A}_k} \|x_k^a - \hat{x}_k^a\|_\infty \leq H, \quad \forall k \in \mathbb{N}.$$

A running example: dynamic-max consensus in open networks (4)

Consider the DMC protocol for open networks:

$$x_k^i = \begin{cases} \max\limits_{j \in \mathcal{N}_{k-1}^i} \left\{ x_{k-1}^j - \alpha, u_k^i \right\} & \text{if } i \in \mathcal{R}_k, \\ u_k^i & \text{if } i \in \mathcal{A}_k. \end{cases}, \qquad \text{and assume} \qquad \begin{cases} |u_k^i - u_{k-1}^i| \leq \Pi, \\ |\overline{u}_k - \underline{u}_k| \leq \Xi. \end{cases}$$

- $B = (\overline{\delta}_k + 1)\alpha$: the maximum variation of the TPI is given by $\delta_k \alpha$ plus a further contribution given by the change of the maximum input, upper bounded by $\Pi < \alpha$.
- $\Lambda = (\overline{\delta}_k + 1)\alpha$;
- $H = \Xi$: since arriving agents initialize their state to their input, and the inputs are assumed to lie within a range of size Ξ.

Theorem 1: Convergence of paracontractive OMASs

Given an OMAS, if:

a) it is (Γ, T)-paracontractive w.r.t. $\|\cdot\|_\infty$ and $\gamma \in (0,1)$;
b) it is Λ-slowly expansive w.r.t. $\|\cdot\|_\infty$;
c) it admits a TPI with bounded variation with $B \geq 0$;
d) its arrival process is bounded with $H \geq 0$;
e) it has dwell time $\Upsilon \geq 0$.

and if $\Upsilon \geq T - 1$, then the TPI is globally asymptotically open stable with radius

$$R = \rho + \min\{T-1, 1\}(\Lambda + B).$$

where

$$\rho = \max\left\{\frac{(T-1)\Lambda + (2T-1)B}{1-\gamma}, \Gamma + TB, H\right\}.$$

Special cases

We now discuss how the stability radius simplifies in some special cases:

- If the OMAS is contractive at each step ($T = 1$, $\Gamma = 0, \Lambda = 0$) then:

$$R = \max\left\{\frac{B}{1-\gamma}, H\right\}.$$

- If further the OMAS is closed ($H = 0$) time-invariant and autonomous ($B = 0$)

$$R = 0.$$

Assumptions

The diameter of the network is bounded by a constant $\overline{\delta} > 0$, i.e., $\delta_k \leq \overline{\delta}, \quad \forall k \geq 0$.

Theorem 2: Open stability of ODMC Protocol

Consider an OMAS executing the ODMC Protocol previously presented under Assumption 1. If the protocol is designed with $\alpha > \Pi$, then the OMAS is open stable with radius R as in Theorem 1 where

$$T = \overline{\delta} + 1, \Gamma = (\overline{\delta} + 1)\alpha, \qquad \Gamma = \Lambda = B = (\overline{\delta} + 1)\alpha, \qquad H = \Xi, \qquad \gamma = \max\{0, \frac{\overline{y}_0 - \overline{u}_1 - \beta - (\Upsilon - \overline{\delta})\alpha}{\|y_0 - \hat{y}_0\|_\infty}\}.$$

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Simulation set-up

The network is initialized as follows:

- The network initially consists of $n_0 = 100$ agents;
- The initial graph is randomly generated with diameter $\delta_0 = 5$ (and we assume that $\delta_k \leq 5 := \overline{\delta}$);
- The initial state of the agents is chosen uniformly at random in the interval $[10, 11]$;
- The inputs are initialized in the interval $[0, 1]$ with bounded variation $\Pi = 0.01$.

During the execution of the algorithm:

- Agents may join and leave with probabilities $p_k^{join}, p_k^{leave} \in [0, 1]$ according to:

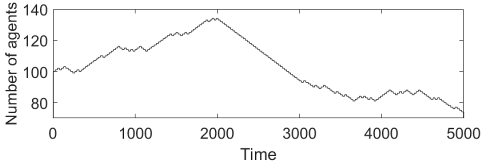
$$[p_k^{join}, p_k^{leave}] = \begin{cases} [0.4, 0.1] & \text{if } k \leq 2000, \\ [0.1, 0.8] & \text{if } k \in (2000, 3000], \\ [0.3, 0.3] & \text{if } k > 3000; \end{cases}$$

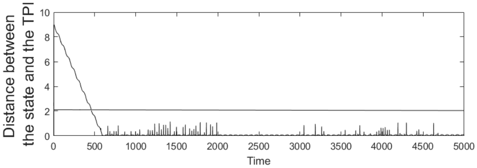
- The inputs varies according to:

$$u_k^i = u_{k-1}^i + \Pi \sin\left(\frac{k}{50}\right);$$

Simulation results

The stability radius is according to Theorem 1 and 2 equal to $R = 2.05$.





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Presentation of a novel algorithm to solve the "*max-tracking problem*", also knwon as the "*dynamic max-consensus problem,*" in Open MAS which falls into the class of "*paracontractive*" open multi-agent systems

Future (ongoing) work: Distributed tracking of graph parameters in Open MAS

Diego Deplano, Mauro Franceschelli, Alessandro Giua, "Distributed Tracking of Network Size, Diameter, Radius, and Node Eccentricities in Open Multi-Agent Systems", under review.

Stability of Paracontractive Open Multi-Agent Systems

Thank you for your attention!

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